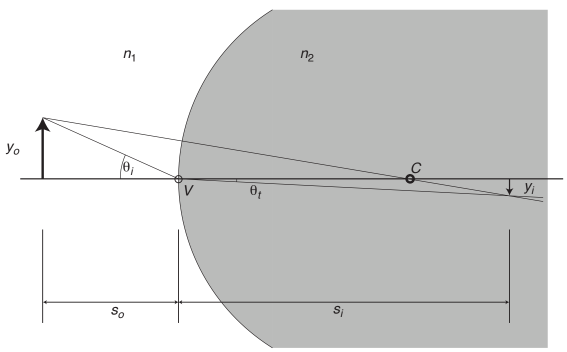
1. Spherical Interface Between Two Media (4 pts)

A spherical interface with radius $R > 0$ separates two continuous media with indices n_1 and n_2, as shown in Hecht Fig. 5.6. Using Snell's law and the paraxial approximation, show that the transverse magnification at the interface is given by

$$M_T = -\frac{n_1 s_i}{n_2 s_o} .$$

Start with a sketch in which you construct the image, within medium 2, of an object of height y_o, located within medium 1. For this construction, use the location C of the center of the curved interface, which is a distance R away from the vertex. Analytically determine the height of the image, y_i, and thereby M_T.



Snell's law, $n_1 \sin\theta_i = n_2 \sin\theta_t$, becomes $n_1\theta_i = n_2\theta_t$ in the paraxial approximation. Referring to the geometry in the sketch, $y_o/s_o = \tan\theta_i \simeq \theta_i$ and $-y_i/s_i = \tan\theta_t \simeq \theta_t$.

Then, $M_T = y_i/y_o = -\dfrac{n_1 s_i}{n_2 s_o}$.

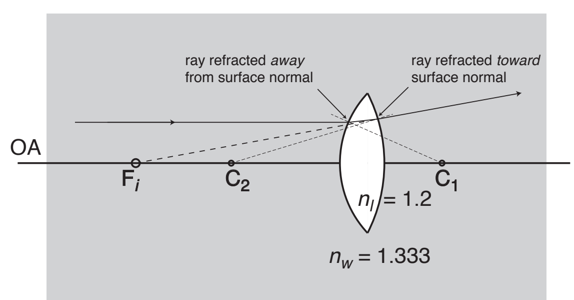
2. Lens in Air and in Water (4 pts)

A thin biconvex lens ($n_l = 1.2$) has radii $R_1 = 100$ mm and $R_2 = 150$ mm. Compute its focal lengths f

(a) in air and (b) in water ($n_w = 1.33$)

in the paraxial approximation and discuss for both media if the lens is converging or diverging.

To rationalize the result obtained in (b), sketch the lens at a scale of 1:5 and trace one ray of a bundle incident parallel to the optical axis ($s_o = \infty$) showing that the refraction at the front and back of the lens faces is consistent with Snell's law given the magnitudes of the indices ($n_l < n_w$).



From $\dfrac{n_m}{s_{o1}} + \dfrac{n_m}{s_{i2}} = (n_l - n_m)(1/R_1 - 1/R_2)$, we obtain $s_{o1}\big|_{s_{i2}\to\infty} = f_o = \dfrac{n_m \cdot R_1 \cdot R_2}{(R_2 - R_1)(n_l - n_m)}$.

Putting numbers (note that the numerical value of $R_2 < 0$, because its center lies to the left of the vertex), we obtain (a) $f = 300$ mm in air and (b) $f = -600$ mm in water. (It was shown in class that $f_o = f_i = f$.) For (a), $f > 0$ implies that the lens converges light rays. In (b), the lens is diverging, because its index is *lower* than that of the surrounding medium (water).

3. Lens Separating Two Media **(4 pts)**

Using the same argument as for the derivation of Hecht Eqn. (5.14) with $d = 0$ (thin lens), but now for two distinct media on both sides of the lens (indices n_m and n_m'), we obtain

$$\frac{n_m}{s_{o1}} + \frac{n'_m}{s_{i2}} = \frac{n_l - n_m}{R_1} + \frac{n'_m - n_l}{R_2}$$

From this eqn., it follows that

$$f_o^{-1} = \frac{1}{s_{o1}\big|_{s_{i2}\to\infty}} = \frac{n_l - n_m}{n'_m \cdot R_1} + \frac{n'_m - n_l}{n'_m \cdot R_2},$$

leading to ($n_m = 1$, $n_l = 1.2$, $n_m' = 1.333$, $R_1 = 100$ mm, $R_2 = 150$ mm): $f_o = 461.6$ mm

and

$$f_i^{-1} = \frac{1}{s_{i2}\big|_{s_{o1}\to\infty}} = \frac{n_l - n_m}{n_m \cdot R_1} + \frac{n'_m - n_l}{n_m \cdot R_2}\,;\ f_i = 346.2 \text{ mm}$$

4. Imaging the Horse **(3 pts)**

The horse sketched in Hecht Fig. 5.26 is 2.25 m tall and stands with its nose 15 m and its tail 17.5 m from the plane of a thin lens ($f_1 = 3$ m).

(a) Determine the distance s_i of the image of the horse's nose and M_T at that position.
Of which type is the image, and what is its orientation (with respect to the optical axis)?

(b) Where is the image of the tail located, and what is M_L at that position?

(a) $$\frac{1}{s_i} = \frac{1}{f} - \frac{1}{s_o} \rightarrow s_i = \left(1/(3\text{ m}) - 1/(15\text{ m})\right)^{-1} = +3.75\text{ m}$$

$M_T = -s_i/s_o = -3.75\text{ m}/(15\text{ m}) = -0.25$.

Because $s_i > 0$, the image is *real*. Because $M_T < 0$, the image is inverted.

(b) $$\frac{1}{s_i} = \frac{1}{f} - \frac{1}{s_o} \rightarrow s_i = \left(1/(3\text{ m}) - 1/(17.5\text{ m})\right)^{-1} = +3.621\text{ m}$$

$$M_L = \frac{\Delta s_i}{\Delta s_o} = \frac{0.129\text{ m}}{2.5\text{ m}} = 0.052$$

5. Image Blur **(3 pts)**

A 4 mm long piece of thin wire in the object plane 60 cm from a thin lens is well-focussed on a screen where the image is 2 mm long.

(a) Use the paraxial approximation to determine the focal length of the lens.

If the screen is moved 10 mm further away from the lens, the image is blurred to a width of 0.8 mm.

(b) Imaging a source point on the optical axis, determine the diameter of the lens.

5.27 $1/s_o + 1/s_i = 1/f$ and $M_T = -s_i/s_o = -1/2$ hence $1/s_o + 2/s_o = 1/f$ but $s_o = 60.0$ cm, hence $f = 20.0$ cm; draw a ray cone from an axial image point, it enters the edges of the lens and focuses at 30.0 cm and then spreads out beyond to create a blur on the screen; from the geometry $(0.40 \text{ mm})/(10.0 \text{ mm}) = (R/300 \text{ mm})$, $R = 1.2$ cm so the diameter is 2.4 cm.

6. Magnification of a Lens Combination **(4 pts)**

Verify that the transverse magnification M_T of a combination of two thin lenses (lens 1 with f_1 at a distance of d from lens 2 with f_2) as a function of the object and image distances of the combination, s_{o1} and s_{i2}, is given by

$$M_T = \frac{f_1 s_{i2}}{d\left(s_{o1} - f_1\right) - s_{o1} f_1}$$

Is the result independent of f_2?

$$M_{T_1} = -s_{i1}/s_{o1} = -\frac{f_1}{s_{o1} - f_1} \; ; \; M_{T_2} = -s_{i2}/s_{o2} = -\frac{s_{i2}}{d - s_{i1}}$$

$$M_T = M_{T_1} \cdot M_{T_2} = \frac{f_1 s_{i2}}{\left(s_{o1} - f_1\right)\left(d - s_{i1}\right)} = \frac{f_1 s_{i2}}{\left(s_{o1} - f_1\right)d - s_{o1} f_1}$$

s_{i2} depends on f_2, so M_T depends implicitly on f_2.